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Two-Commodity Inventory System for Base-Stock Policy with Service Facility

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TWO-COMMODITY INVENTORY SYSTEM FOR BASE-STOCK POLICY WITH SERVICE FACILITY

Strictly as per the compliance and regulations of :

Two-Commodity Inventory System for Base-Stock Policy with Service Facility

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Abstract - This article considers a two-commodity continuous review inventory system at a service facility, wherein an item demanded by a customer is issued after performing service on the item. The service facility is assumed to have a finite waiting hall. The arrival time points of customers form a Poisson process. A customer with probability p and a negative customer with probability $q=(1-p),(0\leq p\leq 1)$. An ordinary customer, on arrival, joins the queue and the negative customer does not join the queue and takes away one waiting customer if any. The life time of each item and service time are assumed to have independent exponential distribution. The joint probability distribution of the number of customers in the system and the inventory level is obtained in the steady state. Various system performance measures in the steady state are derived. The results are illustrated numerically.

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I. Introduction

The $(S-1,S)$ or one-to-one policies are usually implemented for inventory systems stocking expensive, slow moving items. Analysis of continuous review perishable inventory systems with positive lead times under $(S-1,S)$ policy have been carried out by Schmidt and Nahmias (1985), Pal (1989) and Kalpakam and Sapna (1995) and (1996). In all these models, whenever the inventory level drops by one unit, either due to a demand or a failure, an order for one item is placed. Kalpakam and Arivarignan (1998) dealt with a $(S-1,S)$ system with renewal demands for non-perishable items. Kalpakam and Shanthi (2000) have considered modified base stock policy and random supply quantity. Sren Gled Johansen (2005) has considered base-stock policies for the lost sales inventory system with Poisson demand and Erlangian lead times.

Krishnamoorthy et al. (1994) considered a two-commodity continuous review inventory system without lead time. In their model, each demand is for one unit of first commodity or one unit of second commodity or one unit of each commodity with prefixed probabilities. Krishnamoorthy and Varghese (1994) considered a two-commodity inventory problem without leadtime and with Markov shift in demand for the type of commodity namely "commodity-1", "commodity-2" or "both commodity". Yadavalli et al. (2006) have considered a two commodity inventory system with Poisson demands. It is further assumed that the demand for the first commodity require the one unit of second commodity in addition to the first commodity with probability p_1. Similarly, the demand for the second commodity require the one unit of first commodity in addition to the second commodity with probability p_2. Yadavalli et al. (2004) have considered a two commodity inventory system with individual and joint ordering policies.

In most of the inventory models considered in the literature, the demanded items are directly issued from the stock, if available. The demands that occur during stock-out period are either not satisfied (lost sales case) or satisfied only on receipt of the ordered items (backlog case). In the later case, it is assumed that either all (full backlogging) or any prefixed number of demands (partial backlogging) that occurred during stock-out period are satisfied. For review of these works see Nahmias (1982), Raafat (1981), Kalpakam and Arivarignan (1990), Elango and Arivarignan (2003) and Liu and Yang (1999).

But in the case of inventories maintained at service facilities, the demanded items are issued to the customers only after some service is performed on it. In this situation the items are issued not at the time of demand but after a random time of service from the epoch of demand. This forces the formation of queues in these models, which in turn necessitates the study of both inventory level and queue length joint distribution. Berman, Kaplan and Shimshank (1993) have considered an inventory management system at a service facility which uses one item of inventory for each service provided. They assumed that both demand and service times are deterministic and constant, as such queues can from only during stock out periods. They determined optimal order quantity that minimizes the total cost rate.

Berman and Kim (1999) analyzed a problem in stochastic environment where customers arrive at service facilities according to a Poisson process and the service times are exponentially distributed with mean inter-arrival time (assumed to be greater than the mean service time) and each service requires one item from

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inventory. The main result of their work is that under both the discounted cost case and the average cost case, the optimal policy of both the finite and infinite time horizon problem is a threshold ordering policy. The optimal policy in Berman and Kim (1999) is derived given that the order quantity is known. A logically related model was studied by He, Jewkes and Buzacott [9], who analyzed a Markovian inventory-production system, where customer demands arrive at a workshop and are processed by a single machine in batch sizes of one. Berman and Sapna (2000) studied extensively an inventory control problem at a service facility that uses one item of inventory distributed service times and zero lead times. They analyzed the system with the restriction that the waiting space is finite. Under a specified cost structure, they derived the optimal ordering quantity that minimizes the long-run expected cost rate.

Elango (2001) has considered a Markovian inventory system with instantaneous supply of orders at a service facility. The service time is assumed to have exponential distribution with parameter depending on the number of waiting customers. Arivarignan et al. (2002) have extended this model to include exponential inventory system in which the size of the space for the waiting customers is infinite. Arivarignan and Sivakumar (2003) have considered an inventory system with arbitrarily distributed demand, exponential service time and exponential lead time. Finally, Sivakumar et al. (2005) considered a two-commodity perishable inventory system under continuous review at a service facility with a finite waiting room.

In this paper we have considered a $(S-1,S)$ policy for two-commodity stochastic inventory system under continuous review at a service facility with a finite waiting room for customers. The customers arriving to the service station are classified as ordinary (positive or regular) and negative customers. The arrival of ordinary customers to the service station increase the queue length by one and the arrival of negative customer to the service station causes one ordinary customer to be removed if anyone is present.

In the real life situation, the sale agencies deal with two different items with high cost like email server and data server, refrigerator and washing machine etc.,. Keeping them in stock for sales purpose is high risk but yield high profit, wherein the waiting customers may be wooed or taken away by new arriving customers from a large population, many companies look for the prospective customers at others' sales centres. This motivates the researcher to consider the negative customer at a service facility for two commodities with $(S-1,S)$ policy.

The remainder of this paper is organized as follows. In Section 2, we present the mathematical model and the notations. Analysis of the model and the steady state solution are given in Section 3. In Section 4, we derive various measures of system performance in steady state. The total expected cost rate is calculated in Section 5. Our numerical study is presented in Section 6. Section 7 has concluding remarks.

II. Problem Formulation

Consider a two commodity stochastic inventory system with service facility in which the items are delivered to the demanding customers. The demand is for single item per customer. The maximum capacity of the i-th commodity is $S_i (i=1,2)$ units and the waiting hall space is M.

The following assumptions are made:

- The arrival times of customers form a Poisson process with parameter λ. The probability that an ordinary customer is p and a negative is $q(=1-p)$.
- The removal rule adopted in this paper is RCE (Removal of a customer at the end), i.e., arrival of a negative customer removes only a customer at the end including the one who is receiving the service at the time of arrival of a negative customer. The arrival of a negative customer has no effect to the empty service station.
- The demands occur either one unit of first commodity or one unit of second commodity or one unit of each commodity and the service time for each demand follows a negative exponential distribution with parameters γ_1, γ_2 and γ_{12} respectively.
- A one-to-one ordering policy is adopted. According to this policy, orders are placed for one unit of i-th commodity, as and when the inventory level of i-th commodity drops due to a demand $(i=1,2)$.
- The lead times of the reorders for the i-th commodity are assumed to be distributed as a negative exponential with parameter μ_i, $i=1,2$.
- The demands that occur during stock-out periods are lost.

A. Notations

$[A]_{ij}$: The element/submatirx at (i,j) th position of A.

0 : Zero matrix.

1 : An identity matrix.

e : A column vector of 1s appropriate dimension.

$$\delta_{ij} = \begin{cases} 1, & if\ i=j \\ 0, & otherwise. \end{cases}$$

$$\bar{\delta}_{ij} = (1-\delta_{ij})$$

$$E_1 = \{0,1,...,S_1\}$$

$$E_2 = \{0,1,...,S_2\}$$

$$E_3 = \{0,1,\ldots,M\}$$

$E = E_1 \times E_2 \times E_3$

$$[G_1]_{mn} = \begin{cases} \gamma_1 + \delta_{k0}\gamma_{12}, & n = m-1, \quad m = 1,2,\dots,M \\ 0, & otherwise. \end{cases}$$

$$[G_2]_{mn} = \begin{cases} \gamma_2 + \delta_{i0}\gamma_{12}, & n = m-1, \quad m = 1,2,\dots,M \\ 0, & otherwise. \end{cases}$$

$$[G_{12}]_{mn} = \begin{cases} \gamma_{12}, & n = m, \quad m = 1,2,\dots,M \\ 0, & otherwise. \end{cases}$$

$$[P_{ik}]_{mn} = \begin{cases} -(p\lambda + (S_1 - i)\mu_1 + (S_2 - k)\mu_2), & n = m, \quad m = 0 \\ -(\lambda + \bar{\delta}_{0i}\gamma_1 + \bar{\delta}_{0k}\gamma_2 + (1-\delta_{0i}\delta_{0k})\gamma_{12} + (S_1 - i)\mu_1 + (S_2 - k)\mu_2), & n = m, \quad m = 1,2,\dots,M-1 \\ -(q\lambda + \bar{\delta}_{0i}\gamma_1 + \bar{\delta}_{0k}\gamma_2 + (1-\delta_{0i}\delta_{0k})\gamma_{12} + (S_1 - i)\mu_1 + (S_2 - k)\mu_2), & n = m, \quad m = M \\ 0, & otherwise. \end{cases}$$

where $i = 0,1,\dots,S_1$ and $k = 0,1,\dots,S_2$

$$[M_{S_1 - i}]_{mn} = \begin{cases} (S_1 - i)\mu_1, & n = m, \quad m = 0,1,\dots,M \\ 0, & otherwise. \end{cases}$$

$$[U_{S_2 - k}]_{mn} = \begin{cases} (S_2 - k)\mu_2, & n = m, \quad m = 0,1,\dots,M \\ 0, & otherwise. \end{cases}$$

III. Analysis

Let $L_i(t)$ denote the inventory level of i-th commodity and $X(t)$ denote the number of customers (waiting and being served) in the system, at time t. From the assumptions made on the input and output processes, it may be shown that the triplet

$$(L_1, L_2, X) = \{(L_1(t), L_2(t), X(t),), t \geq 0\},$$

on the state space E, is a Markov process. The infinitesimal generator of this process,

$$A = (a((i,k,m),(j,l,n))), \quad (i,k,m),(j,l,n) \in E$$

can be obtained by using the following arguments:

- The arrival of an ordinary customer makes a transition from (i,k,m) to $(i,k,m+1), i = 0,1,\dots,S_1$, $k = 0,1,\dots,S_2$, $m = 0,1,\dots,M-1$ with intensity of transition $p\lambda$, and the arrival of a negative customer makes a transition from (i,k,m) to $(i,k,m-1)$, $i = 0,1,\dots,S_1$, $k = 0,1,\dots,S_2$, $m = 1,2,\dots,M$ with intensity of transition $q\lambda$, $q = 1-p$. The arrival of a negative customer has no effect on empty service station.

- The service completion involving the first commodity forces one customer to leave the system and a decrease of one item in the inventory level of the first commodity. Thus a transition takes place from (i,k,m) to $(i-1,k,m-1)$, $i = 1,2,\dots,S_1$, $k = 0,1,\dots,S_2$, $m = 1,2,\dots,M$ with intensity γ_1.

- Similarly, a service completion involving the second commodity forces one customer to leave the system and a decrease of one item in the inventory level of the second commodity. Thus a transition takes place from state (i,k,m) to $(i,k-1,m-1)$, $i = 0,1,\dots,S_1$, $k = 1,2,\dots,S_2$, $m = 1,2,\dots,M$ with intensity γ_2.

- A transition from state (i,k,m) to $(i-1,k-1,m-1)$ takes place when a service completion involving both commodity forces one customer to leave the system and a decrease of one item in the inventory level of first and second commodities. The intensity of the transition is γ_{12}, $i = 1,2,\dots,S_1$, $k = 1,2,\dots,S_2$, $m = 1,2,\dots,M$.

- A transition from state (i,k,m) to $(i+1,k,m)$ for $i = 0,1,\dots,S_1 - 1$, $k = 0,1,\dots,S_2$, $m = 0,1,\dots,M$ takes place when a replenishment occurs for the first commodity with intensity μ_1. Similarly, a transition from state (i,k,m) to $(i,k+1,m)$ for $i = 0,1,\dots,S_1$, $k = 0,1,\dots,S_2 - 1$, $m = 0,1,\dots,M$ takes place when a replenishment occurs for the second commodity with intensity μ_2.

- For other transition from (i,k,m) to (j,l,n) except $(i,k,m) \neq (j,l,n)$, the rate is zero.

- Finally, note that

$$a((i,k,m),(i,k,m)) = -\sum_{j}\sum_{l}\sum_{\substack{n \\ (j,l,n)\neq(i,k,m)}} a((i,k,m),(j,l,n)).$$

Hence we have, $a((i,k,m),(j,l,n))$ is given by

$$
\begin{cases}
p\lambda, & n=m+1, & m=0,1,\ldots,M-1\\
 & l=k, & k=0,1,\ldots,S_2\\
 & j=i, & i=0,1,\ldots,S_1\\
q\lambda, & n=m-1, & m=1,2,\ldots,M\\
 & l=k, & k=0,1,\ldots,S_2\\
 & j=i, & i=0,1,\ldots,S_1\\
-(p\lambda+(S_1-i)\mu_1+ & n=m, & m=0\\
(S_2-k)\mu_2), & l=k, & k=0,1,\ldots,S_2\\
 & j=i, & i=0,1,\ldots,S_1\\
-(\lambda+\bar{\delta}_{0i}\gamma_1+\bar{\delta}_{0k}\gamma_2+ & n=m, & m=1,2,\ldots,M-1\\
(1-\delta_{0i}\delta_{0k})\gamma_{12}+ & l=k, & k=0,1,\ldots,S_2\\
(S_1-i)\mu_1+(S_2-k)\mu_2), & j=i, & i=0,1,\ldots,S_1\\
-(q\lambda+\bar{\delta}_{0i}\gamma_1+\bar{\delta}_{0k}\gamma_2+ & n=m, & m=M\\
(1-\delta_{0i}\delta_{0k})\gamma_{12}+ & l=k, & k=0,1,\ldots,S_2\\
(S_1-i)\mu_1+(S_2-k)\mu_2), & j=i, & i=0,1,\ldots,S_1\\
\gamma_1+\delta_{k0}\gamma_{12}, & n=m-1, & m=1,2,\ldots,M\\
 & l=k, & k=0,1,\ldots,S_2\\
 & j=i-1, & i=1,2,\ldots,S_1
\end{cases}
$$

$$
\begin{cases}
\gamma_2+\delta_{i0}\gamma_{12}, & n=m-1, & m=1,2,\ldots,M\\
 & l=k-1, & k=1,2,\ldots,S_2\\
 & j=i, & i=0,1,\ldots,S_1\\
\gamma_{12}, & n=m-1, & m=1,2,\ldots,M\\
 & l=k-1, & k=1,2,\ldots,S_2\\
 & j=i-1, & i=1,2,\ldots,S_1\\
(S_1-i)\mu_1, & n=m, & m=0,1,\ldots,M\\
 & l=k, & k=0,1,\ldots,S_2\\
 & j=i+1, & i=0,1,\ldots,S_1-1\\
(S_2-i)\mu_2, & n=m, & m=0,1,\ldots,M\\
 & l=k+1, & k=0,1,\ldots,S_2-1\\
 & j=i, & i=0,1,\ldots,S_1\\
0, & otherwise. &
\end{cases}
$$

More explicitly,

$$
A=\begin{array}{c}
S_1\\ S_1-1\\ S_1-2\\ \vdots\\ 1\\ 0
\end{array}
\begin{pmatrix}
A_{S_1} & B & & & & \\
C_1 & A_{S_1-1} & B & & & \\
 & C_2 & A_{S_1-2} & B & & \\
 & & \ddots & \ddots & \ddots & \\
 & & & C_{S_1-1} & A_1 & B\\
 & & & & C_{S_1} & A_0
\end{pmatrix}
$$

Where

$$
[B]_{kl}=\begin{cases}
G_1, & l=k, & k=0,1,\ldots,S_2\\
G_{12}, & l=k-1, & k=1,2,\ldots,S_2\\
0, & otherwise
\end{cases}
$$

$$
[C_{S_1-i}]_{kl}=\begin{cases}
M_{S_1-i}, & l=k, & k=0,1,\ldots,S_2\\
0, & otherwise
\end{cases}
$$

$for \quad i=0,1,\ldots,S_1$

$$
[A_i]_{kl}=\begin{cases}
G_2, & l=k-1, & k=1,2,\ldots,S_2\\
U_{S_2-k}, & l=k+1, & k=0,1,\ldots,S_2-1\\
P_{ik}, & l=k, & k=0,1,\ldots,S_2\\
0, & otherwise
\end{cases}
$$

A Steady State Analysis

It can be seen from the structure of A that the homogeneous Markov process $\{(L_1(t),L_2(t),X(t))t\geq 0\}$ on the finite state space E is irreducible, aperiodic and persistent non-null. Hence the limiting distribution of the Markov process exists.

Let $\Pi,$ partitioned as $\Pi=(\Pi^{(S_1)},\Pi^{(S_1-1)},\ldots,\Pi^{(1)},\Pi^{(0)}),$ denote the steady state probability vector of A. That is, Π satisfies

$$\Pi A = 0 \quad and \quad \Pi e = 1 \qquad (1)$$

The components of the vector $\Pi^{(q)}$ $(0 \le q \le S_1)$ are $\Pi^{(q)} = (\pi^{(q,S_2)}, \ldots, \pi^{(q,1)}, \pi^{(q,0)})$, where for $0 \le l \le S_2$, $\pi^{(q,l)} = (\pi^{(q,l,0)}, \pi^{(q,l,1)}, \ldots, \pi^{(q,l,M)})$.

From the structure of A, it is seen that the Markov process under study falls into the class of birth and death process in a Markovian environment as discussed by Gaver et al. (1984). Hence using the same argument, we can calculate the limiting probability vectors. For the sake of completeness, we provide the algorithm here.

Algorithm :

1. Determine recursively the matrices

$$F_0 = A_0$$
$$F_i = A_i + B(-F_{i-1}^{-1})C_{S_1-i+1}, \qquad i = 1,2,\ldots S_1$$

2. Compute recursively the vectors $\Pi^{(i)}$ using

$$\Pi^{(i)} = \Pi^{(i+1)} B(-F_i^{-1}), \qquad i = 0,1,\ldots,S_1 - 1$$

3. Solve the system of equations

$$\Pi^{(S_1)} F_{S_1} = 0$$
$$\sum_{i=0}^{S_1} \Pi^{(i)} e = 1.$$

From the system of equations $\Pi^{(S_1)} F_{S_1} = 0$, vector $\Pi^{(S_1)}$ could be determined uniquely, upto a multiplicative constant. This constant is decided by

$$\Pi^{(i)} = \Pi^{(i+1)} B(-F_i^{-1}), i = 0,1,\ldots,S_1 - 1$$

and $$\sum_{i=0}^{S_1} \Pi^{(i)} e = 1.$$

IV. System Performance Measures

In this section, some performance measures of the system are derived under consideration.

a) Mean Inventory Levels

Let η_1 and η_2 be the average inventory level for the first commodity and the second commodity respectively in the steady state. Then we have,

$$\eta_1 = \sum_{i=1}^{S_1} i \left(\sum_{k=0}^{S_2} \sum_{m=0}^{M} \pi^{(i,k,m)} \right)$$

and

$$\eta_2 = \sum_{k=1}^{S_2} k \left(\sum_{i=0}^{S_1} \sum_{m=0}^{M} \pi^{(i,k,m)} \right)$$

b) Mean Reorder Rates

Let η_3 and η_4 denote the mean reorder rate for the first and second commodities respectively. Then we have,

$$\eta_3 = (\gamma_1 + \gamma_{12}) \sum_{i=1}^{S_1} \sum_{k=0}^{S_2} \sum_{m=1}^{M} \pi^{(i,k,m)}$$

and

$$\eta_4 = (\gamma_2 + \gamma_{12}) \sum_{i=0}^{S_1} \sum_{k=1}^{S_2} \sum_{m=1}^{M} \pi^{(i,k,m)}$$

c) Mean Rate of Arrivals of Negative Customers

Let η_{NC} denote the mean rate of arrivals of negative customers for the system. Then we have,

$$\eta_{NC} = \sum_{i=0}^{S_1} \sum_{k=0}^{S_2} q\lambda \left(\sum_{m=1}^{M} \pi^{(i,k,m)} \right)$$

d) Mean Balking Rate

Let η_B denote the mean balking rate. Then we have,

$$\eta_B = p\lambda \sum_{i=0}^{S_1} \sum_{k=0}^{S_2} \pi^{(i,k,M)}$$

e) Mean Waiting time

Let $\overline{W}$ denote the mean waiting time of the customers. Then, by Little's formula

$$\overline{W} = \frac{\Gamma}{\lambda_a}$$

where, $$\Gamma = \sum_{m=1}^{M} m \left(\sum_{i=0}^{S_1} \sum_{k=0}^{S_2} \pi^{(i,k,m)} \right).$$

where λ_a denotes the expected arrival rate which is given by

$$\lambda_a = p\lambda \sum_{i=0}^{S_1}\sum_{k=0}^{S_2}\sum_{m=0}^{M-1}\pi^{(i,k,m)}.$$

V. Cost Optimization

In order to compute the total expected cost per unit time, we introduce the following notations:

c_{h_1} : The inventory holding cost per unit item per unit time for I-commodity.

c_{h_2} : The inventory holding cost per unit item per unit time for II-commodity.

c_{s_1} : The setup cost per order for I-commodity.

c_{s_2} : The setup cost per order for II-commodity.

c_N : Cost of loss per unit time due to arrival of a negative customer.

c_w : Waiting time cost of a customer per unit time.

c_B : Balking cost per customer per unit time.

Then the long-run expected cost rate is given by

$$TC(S_1,S_2,M) = c_{h_1}\eta_1 + c_{h_2}\eta_2 + c_{s_1}\eta_3 + c_{s_2}\eta_4 + c_N\eta_{NC} + c_w\overline{W} + c_B\eta_B.$$

Substituting $\eta's$ and $\overline{W}$ into the above equation, we obtain

$$TC(S_1,S_2,M) = c_{h_1}\left(\sum_{i=1}^{S_1} i\left(\sum_{k=0}^{S_2}\sum_{m=0}^{M}\pi^{(i,k,m)}\right)\right) + c_{h_2}\left(\sum_{k=1}^{S_2} k\left(\sum_{i=0}^{S_1}\sum_{m=0}^{M}\pi^{(i,k,m)}\right)\right)$$

$$+c_{s_1}\left((\gamma_1+\gamma_{12})\sum_{i=1}^{S_1}\sum_{k=0}^{S_2}\sum_{m=1}^{M}\pi^{(i,k,m)}\right) + c_{s_2}\left((\gamma_2+\gamma_{12})\sum_{i=0}^{S_1}\sum_{k=1}^{S_2}\sum_{m=1}^{M}\pi^{(i,k,m)}\right)$$

$$+c_N\left(\sum_{i=0}^{S_1}\sum_{k=0}^{S_2} q\lambda\left(\sum_{m=1}^{M}\pi^{(i,k,m)}\right)\right) + c_w\left(\frac{\Gamma}{\lambda_a}\right) + c_B\left(p\lambda\sum_{i=0}^{S_1}\sum_{k=0}^{S_2}\pi^{(i,k,M)}\right)$$

Due to the complex form of the limiting distribution, it is difficult to discuss the properties of the cost fuction analytically. Hence, a detailed computational study of the cost function is carried out in the next section.

VI Numerical Examples

Since we have not shown analytically the convexity of the function $TC(S_1,S_2,M)$ we have explored the behavior of this function by considering it as functions of any two variable by fixing the other one at a constant value.

The *table 1* gives the total expected cost rate for various combinations of S_1 and S_2 when fixed values for other parameters and costs are assumed. They are $M=3$, $\lambda=22$, $\gamma_1=3$, $\gamma_2=5$, $\gamma_{12}=9$, $p=0.7$, $q=0.3$, $\mu_1=1$, $\mu_2=2.1$, $c_{h_1}=6.7$, $c_{h_2}=7$, $c_{s_1}=0.2$, $c_{s_2}=0.5$, $c_N=5$, $c_w=5$, $c_B=0.5$.

Moreover, *Figure 1.* refers the changes of S_1 and S_2 are how to affect the total expected cost rate.

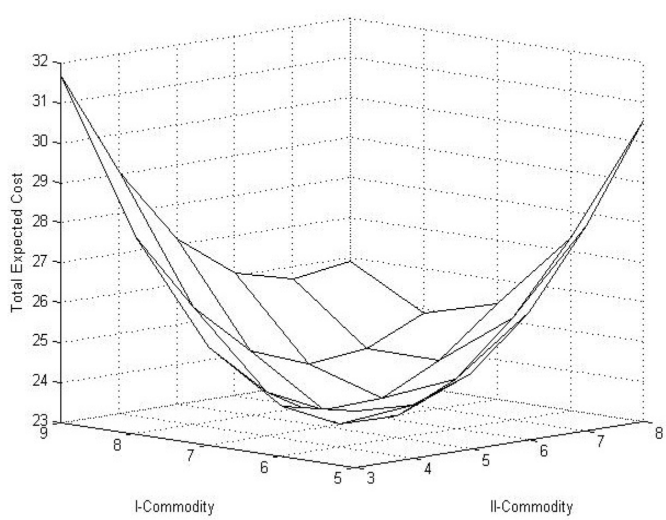

Fig.1. Convexity of the total cost for various combinations of S_1 and S_2.

S_2 / S_1	3	4	5	6	7	8
5	24.3900	24.3208	24.9074	26.1902	28.1159	30.5480
6	**24.2305**	**23.5581**	23.5474	24.2324	25.5464	27.3442
7	25.4291	24.0843	**23.4229**	**23.4859**	**24.1975**	25.3994
8	27.9424	25.9299	24.6207	24.0764	24.2217	**24.8854**
9	31.6487	29.0385	27.1422	26.0501	25.6960	25.8998

Table.1. Total expected cost rate as a function of S_1 and S_2

Let $TC_1(S_1, S_2) = TC(S_1, S_2, 3)$. The values of $TC_1(S_1, S_2)$ are given in the above table. The optimal cost for each S_2 is shown in bold and the optimal cost for each S_1 is underlined. The numerical values shows that $TC_1(S_1, S_2)$ is a convex function in (S_1, S_2) and the (possibly local) optimum occurs at $(S_1, S_2) = (7, 5)$.

The table 2 gives the total expected cost rate for various combinations of S_1 and M. We have assumed constant values for other parameters and costs. Namely, $S_2 = 12$, $\lambda = 11.7$, $\gamma_1 = 3.5$, $\gamma_2 = 2.5$, $\gamma_{12} = 4.5$, $p = 0.7$, $q = 0.3$, $\mu_1 = 0.5$, $\mu_2 = 1.05$, $c_{h_1} = 4$, $c_{h_2} = 3$, $c_{s_1} = 1$, $c_{s_2} = 2$, $c_N = 0.2$, $c_w = 15$, $c_B = 15.5$.

S_1 / M	8	9	10	11	12
4	23.52099	21.82993	20.95269	20.94535	21.81381
5	23.13975	21.64211	20.87712	**20.90479**	**21.74334**
6	22.92739	21.55864	**20.87555**	20.94015	21.77652
7	**22.77410**	**21.53674**	20.91023	21.00445	21.84678
8	22.82032	21.54629	20.95764	21.07334	21.92291

Table.2. Total expected cost rate as a function of S_1 and M

Let $TC_2(S_1, M) = TC(S_1, 12, M)$. The values of $TC_2(S_1, M)$ are given in the above table. The optimal cost for each S_1 is shown in bold and the optimal cost for each M is underlined. The numerical values shows that $TC_2(S_1, M)$ is a convex function in (M, S_1) and the (possibly local) optimum occurs at $(M, S_1) = (6, 10)$.

The table 3 gives the total expected cost rate for various combinations of S_2 and M. We have assumed constant values for other parameters and costs. Namely, $S_1 = 6$, $\lambda = 6$, $\gamma_1 = 3.5$, $\gamma_2 = 2.5$, $\gamma_{12} = 4.5$, $p = 0.7$, $q = 0.3$, $\mu_1 = 0.5$, $\mu_2 = 1.05$, $c_{h_1} = 4$, $c_{h_2} = 5$, $c_{s_1} = 1$, $c_{s_2} = 1.5$, $c_N = 0.6$, $c_w = 15$, $c_B = 16.5$.

M / S_2	3	4	5	6	7
2	17.97380	17.43625	17.25528	17.26311	17.36521
3	16.99332	16.66086	16.57168	16.59243	16.65523
4	**16.72198**	**16.52336**	**16.48977**	**16.51536**	**16.55491**
5	17.05421	16.91763	16.90427	16.92590	16.95070
6	17.81789	17.70835	17.69908	17.71507	17.73156

Table.3. Total expected cost rate as a function of S_2 and M

Let $TC_3(S_2, M) = TC(6, S_2, M)$. The values of $TC_3(S_2, M)$ are given in the above table. The optimal cost for each M is shown in bold and the optimal cost for each S_2 is underlined. The numerical values shows that $TC_3(S_2, M)$ is a convex function in (S_2, M) and the (possibly local) optimum occurs at $(S_2, M) = (4, 5)$.

In table 4 the effect of service rates γ_1 and γ_2 on the optimal values (S_1, S_2) and the corresponding total expected cost rate are studied by fixing the parameters and costs $M = 3$, $\lambda = 22$, $\gamma_{12} = 8$, $p = 0.7$, $q = 0.3$, $\mu_1 = 1$, $\mu_2 = 2.1$, $c_{h_1} = 6.7$,

$c_{h_2}=7$, $c_{s_1}=1.2$, $c_{s_2}=1.5$, $c_N=5$, $c_w=5$, $c_B=0.5$. We observed that the total expected cost rate increase when γ_1 and γ_2 increases.

γ_1 \ γ_2	2.69		3.69		4.69		5.69		6.69	
2.49	7	5	7	5	7	5	7	5	7	6
	5.7310		5.7891		5.9164		25.9582		26.0155	
3.49	8	5	7	5	7	5	7	5	7	5
	25.8860		25.9483		25.9452		26.0038		26.1124	
4.49	8	5	8	5	7	4	7	5	7	5
	25.8614		25.9641		26.0933		26.1189		26.1309	
5.49	8	5	8	5	8	5	7	4	7	4
	25.9715		25.9940		26.0538		26.1418		26.2255	
6.49	8	4	8	4	8	5	8	4	7	4
	25.9017		26.0111		26.1216		26.1513		26.2118	

Table.4. Effect of service rates γ_1 and γ_2 on optimal values

Table 5 illustrates the impact of service rates γ_1 and γ_{12} on the optimal values (S_1,S_2) and the corresponding total expected cost rate when $M=3$, $\lambda=20$, $\gamma_2=4.5$, $p=0.7$, $q=0.3$, $\mu_1=1$, $\mu_2=2.1$, $c_{h_1}=7$, $c_{h_2}=8$, $c_{s_1}=2$, $c_{s_2}=1.8$, $c_N=5$, $c_w=5$, $c_B=0.5$. We observed that the total expected cost rate increase when γ_1 and γ_{12} increases.

γ_1 \ γ_{12}	4.5		5.0		5.5		6.0		6.5	
2.5	6	5	6	5	6	5	6	5	6	4
	26.2447		26.2717		26.2962		26.3564		26.4714	
3.0	6	5	6	5	6	5	6	4	6	4
	26.2100		26.2579		26.3601		26.3747		26.3912	
3.5	6	5	6	4	6	4	6	4	6	4
	26.2636		26.2850		26.2896		26.3265		26.3884	
4.0	6	4	6	4	6	4	6	4	7	5
	26.1900		26.2173		26.2733		26.3512		26.4093	
4.5	6	4	6	4	6	4	7	5	7	4
	26.1592		26.2329		26.3255		26.4041		26.4237	

Table.5. Effect of service rates γ_1 and γ_{12} on optimal values

Table 6 illustrates the impact of service rates γ_2 and γ_{12} on the optimal values (S_1,S_2) and the corresponding total expected cost rate when $M=3$, $\lambda=19$, $\gamma_1=1.5$, $p=0.7$, $q=0.3$, $\mu_1=1$, $\mu_2=2.1$, $c_{h_1}=6.7$, $c_{h_2}=7$, $c_{s_1}=1.2$, $c_{s_2}=1.5$, $c_N=5$, $c_w=5$, $c_B=0.5$.

We observed that the total expected cost rate increase when γ_2 and γ_{12} increases.

γ_{12} \ γ_2	1.5		2.5		3.5		4.5		5.5	
2.5	5	4	5	5	4	4	4	4	4	5
	23.8494		24.0158		24.0678		24.2092		24.4204	
3.5	5	4	5	4	5	5	5	5	5	5
	24.0907		24.4758		24.6172		24.8916		25.2715	
4.5	5	4	5	4	5	4	5	5	5	5
	24.0095		24.1568		24.4138		24.4772		24.6397	
5.5	6	5	6	5	6	5	5	4	5	5
	23.9139		23.9874		24.1867		24.4688		24.5837	
6.5	6	4	6	5	6	5	6	5	6	5
	24.9960		25.4432		25.7337		26.0965		26.4013	

Table.6. Effect of service rates γ_{12} and γ_2 on optimal values

Table 7 illustrates the impact of replenishments rates μ_1 and μ_2 on the optimal values (S_1, S_2) and the corresponding total expected cost rate when $M=3$, $\lambda=19$, $\gamma_1=3.5$, $\gamma_2=2.5$, $\gamma_{12}=3.5$, $p=0.7$, $q=0.3$, $c_{h_1}=6.7$, $c_{h_2}=7$, $c_{s_1}=1.2$, $c_{s_2}=1.5$, $c_N=5$, $c_w=5$, $c_B=0.5$. We observed that the total expected cost rate decrease when μ_1 and μ_2 increases.

μ_1 \ μ_2	1.7		1.8		1.9		2.0		2.1	
0.6	9	7	9	6	9	6	9	6	9	6
	27.6291		27.6314		27.4658		27.3444		27.2590	
0.7	8	6	8	6	8	6	8	6	8	6
	26.8882		26.7375		26.6289		26.5547		26.5083	
0.8	7	6	7	6	7	6	7	6	7	6
	26.1883		26.1155		26.0735		26.0563		26.0587	
0.9	7	6	7	6	7	6	7	6	7	6
	25.8493		25.7166		25.6196		25.5519		25.5079	
1.0	6	6	6	6	6	6	6	5	6	5
	25.4296		25.3894		25.3737		25.3516		25.2388	

Table.7. Effect of service rates μ_1 and μ_2 on optimal values

In table 8 the impact of holding costs c_{s_1} and c_{s_2} on the optimal values (S_1, S_2) and the corresponding total expected cost rate are studied by fixing the parameters and costs $M=3$, $\lambda=22$, $\gamma_1=1.5$, $\gamma_2=2.5$, $\gamma_{12}=8$, $p=0.7$, $q=0.3$, $\mu_1=1$, $\mu_2=2.1$, $c_{h_1}=6.7$, $c_{h_2}=7$, $c_N=5$, $c_w=5$, $c_B=0.5$. We observed that the total expected cost rate increase when c_{s_1} and c_{s_2} increases.

c_{s_2} \ c_{s_1}	1.2		1.3		1.4		1.5		1.6	
0.7	7	5	7	5	7	5	7	5	7	5
	25.0526		25.2241		25.3957		25.5672		25.7388	
0.8	7	5	7	5	7	5	7	5	7	5
	25.0652		25.2368		25.4083		25.5799		25.7514	
0.9	7	5	7	5	7	5	7	5	7	5
	25.0779		25.2494		25.4210		25.5925		25.7641	
1.0	7	5	7	5	7	5	7	5	7	5
	25.0905		25.2621		25.4336		25.6052		25.7767	
1.1	7	5	7	5	7	5	7	5	7	5
	25.1032		25.2747		25.4463		25.6178		25.7894	

Table.8. Effect of setup costs c_{s_1} and c_{s_2} on optimal values

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In table 9 the impact of holding costs c_{h_1} and c_{h_2} on the optimal values (S_1, S_2) and the corresponding total expected cost rate are studied by fixing the parameters and costs $M = 3$, $\lambda = 22$, $\gamma_1 = 1.5$, $\gamma_2 = 2.5$, $\gamma_{12} = 8$, $p = 0.7$, $q = 0.3$, $\mu_1 = 1$, $\mu_2 = 2.1$, $c_{s_1} = 1.2$, $c_{s_2} = 1.5$, $c_N = 5$, $c_w = 5$, $c_B = 0.5$. We observed that the total expected cost rate increase when c_{h_1} and c_{h_2} increases.

c_{h_1} \ c_{h_2}	4		5		6		7		8	
3.5	9	7	9	7	9	7	9	7	9	7
	16.5145		16.7850		17.0556		17.3261		17.5967	
4.5	8	6	8	6	8	6	8	6	8	6
	19.3092		19.6377		19.9662		20.2946		20.6231	
5.5	8	7	8	6	8	6	8	6	8	6
	21.8339		22.2329		22.5614		22.8898		23.2183	
6.5	7	6	7	5	7	5	7	5	7	5
	23.8701		24.3651		24.8469		25.2070		25.5670	
7.5	7	6	7	6	7	6	7	5	7	5
	25.8514		26.3464		26.8414		27.3245		27.6846	

Table.9. Effect of holding costs c_{h_1} and c_{h_2} on optimal values

VII. Conclusion

In this paper, we discussed $(S-1, S)$ policy for two-commodity stochastic inventory system under continuous review at a service facility with finite waiting hall. The customers arriving to the service station are classifed as ordinary (positive or regular) and negative customers. Demands occuring during stock out periods are lost. The limiting distribution is obtained by using the algorithm of Gaver (1984). Various system performance measures are derived in the steady state. The results are illustrated with numerically. The model discussed here is useful in studying a service facility for two commodity inventory system which are slow moving items and the high holding cost.

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